Assignment to 2.5
- Solution -

1. Given are the following data:

$C_f := 98$ — C_f = Fixed cost

$C_v(x) := x^3 - 12x^2 + 60x$ — C_v = Variable cost

$C(x) := C_f + C_v(x)$ — C = Cost

Determine for $x := 0..10$ [x = quantity] the values C(x) and C(x+1) - C(x).

x =	C(x) =	C(x + 1) − C(x) =
0	98	49
1	147	31
2	178	19
3	197	13
4	210	13
5	223	19
6	242	31
7	273	49
8	322	73
9	395	103
10	498	139

2. Determine for $x := 1..10$ and for the function $C(x) := 98 + 60x - 12x^2 + x^3$ the values:

$c_v(x) := \frac{C_v(x)}{x}$ — c_v = variable cost per unit

$c_f(x) := \frac{C_f}{x}$ — c_f = fixed cost per unit

$c(x) := \frac{C(x)}{x}$ — c = total cost per unit

x =	$c_v(x) =$	$c_f(x) =$	$c(x) =$
1	49.00	98.00	147.00
2	40.00	49.00	89.00
3	33.00	32.67	65.67
4	28.00	24.50	52.50
5	25.00	19.60	44.60
6	24.00	16.33	40.33
7	25.00	14.00	39.00
8	28.00	12.25	40.25
9	33.00	10.89	43.89
10	40.00	9.80	49.80

Determine the quantity at which c_v is a minimum and at which c is a minimum.

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Can be seen from the list, or Mathcad's functions are used:

$x := 1$ Estimate

$x_{cvmin} := \text{Minimize}(c_v, x)$ Quantity at which c_v is a minimum

$x_{cvmin} = 6$

$x_{cmin} := \text{Minimize}(c, x)$ Quantity at which c is a minimum

$x_{cmin} = 7$

3. Given are the following data

$C_f := 50000$

$C_v(x) := 7000x - 180x^2 + 2x^3$

$C(x) := C_f + C_v(x)$

At which quantity is the minimum of of the first derivative of C(x), the minimum of c_v(x) and the minimum of c(x)?

$x := x$

$\frac{d}{dx}C(x) \rightarrow 7000 - 360 \cdot x + 6 \cdot x^2$

$C'(x) := \frac{d}{dx}C(x)$

$x := 1$ Estimate

$x_{C'min} := \text{Minimize}(C', x)$ Quantity at which C' is a minimum

$x_{C'min} = 30$

$c_v(x) := \frac{C_v(x)}{x}$

$x_{cvmin} := \text{Minimize}(c_v, x)$ Quantity at which c_v is a minimum

$x_{cvmin} = 45$

$c(x) := \frac{C(x)}{x}$

$x_{cmin} := \text{Minimize}(c, x)$ Quantity at which c is a minimum

$x_{cmin} = 50$

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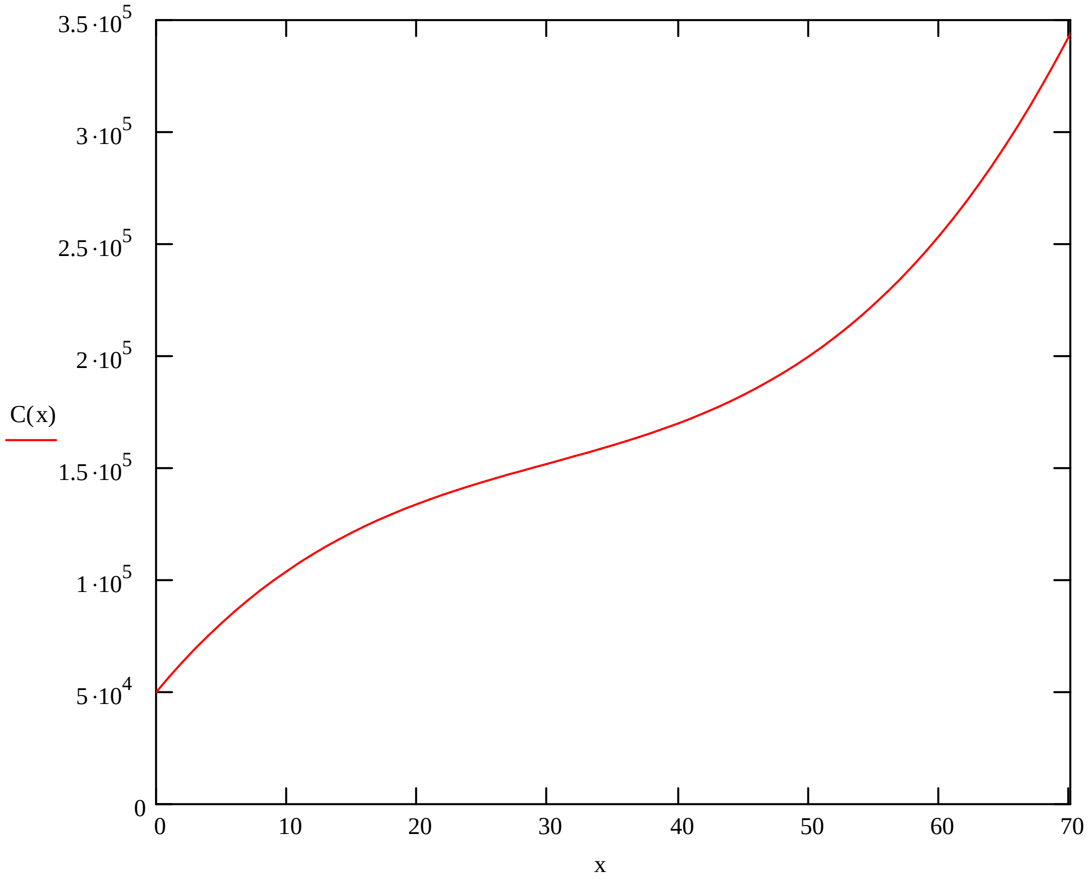
4. For $x := 0..70$ the function from assignment 3 can be illustrated as follows:



How can the minima of C', c_v and c be found graphically?

$$\text{Tangent}(x, x_0) := C(x_0) - C'(x_0) \cdot x_0 + C'(x_0) \cdot x$$

Definitions to make the software solve the problem, not necessary for understanding it.

$$x_0 := x_{C'min}$$

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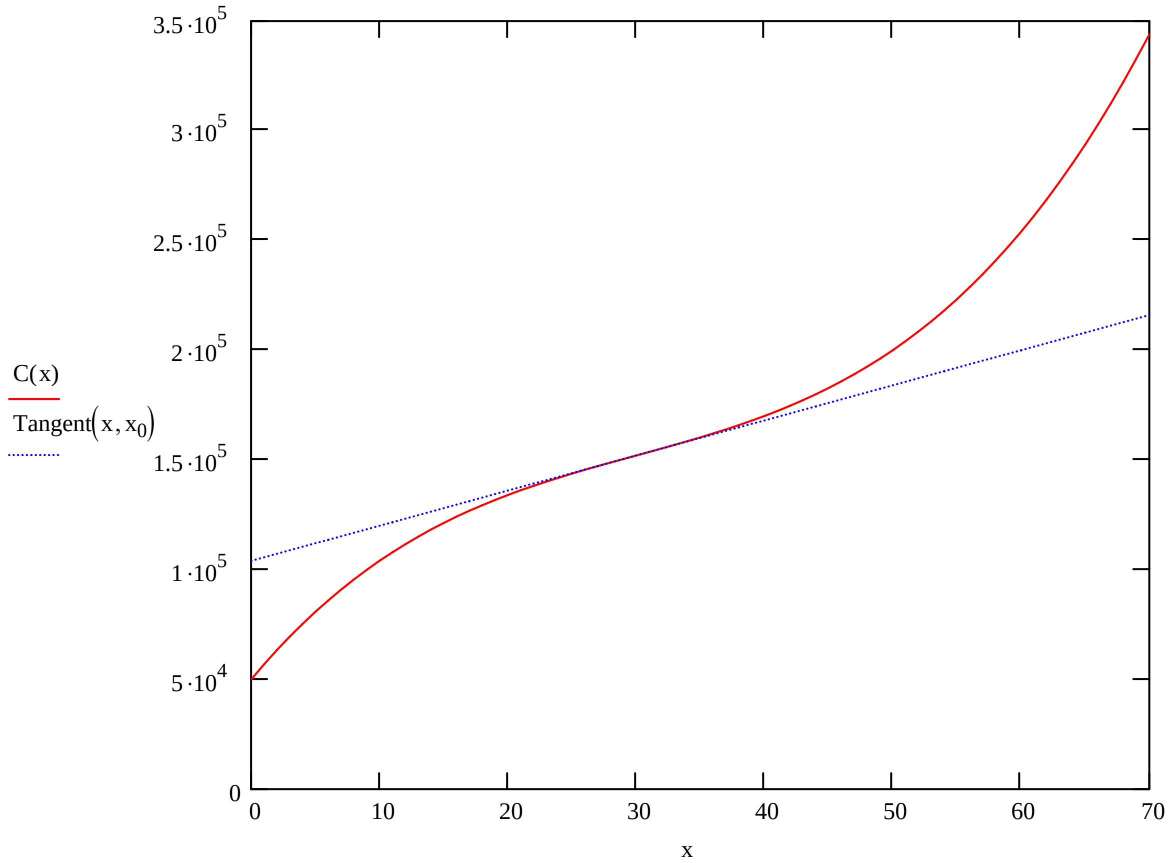
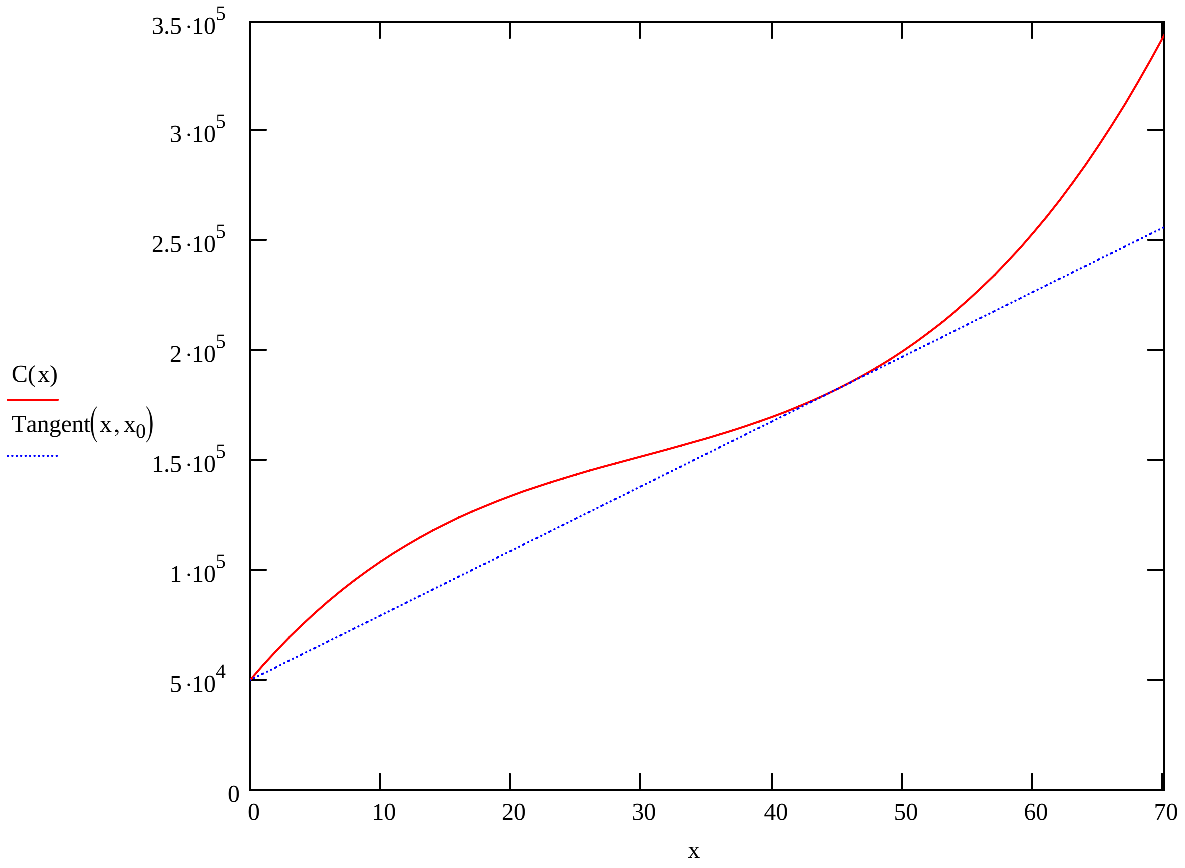
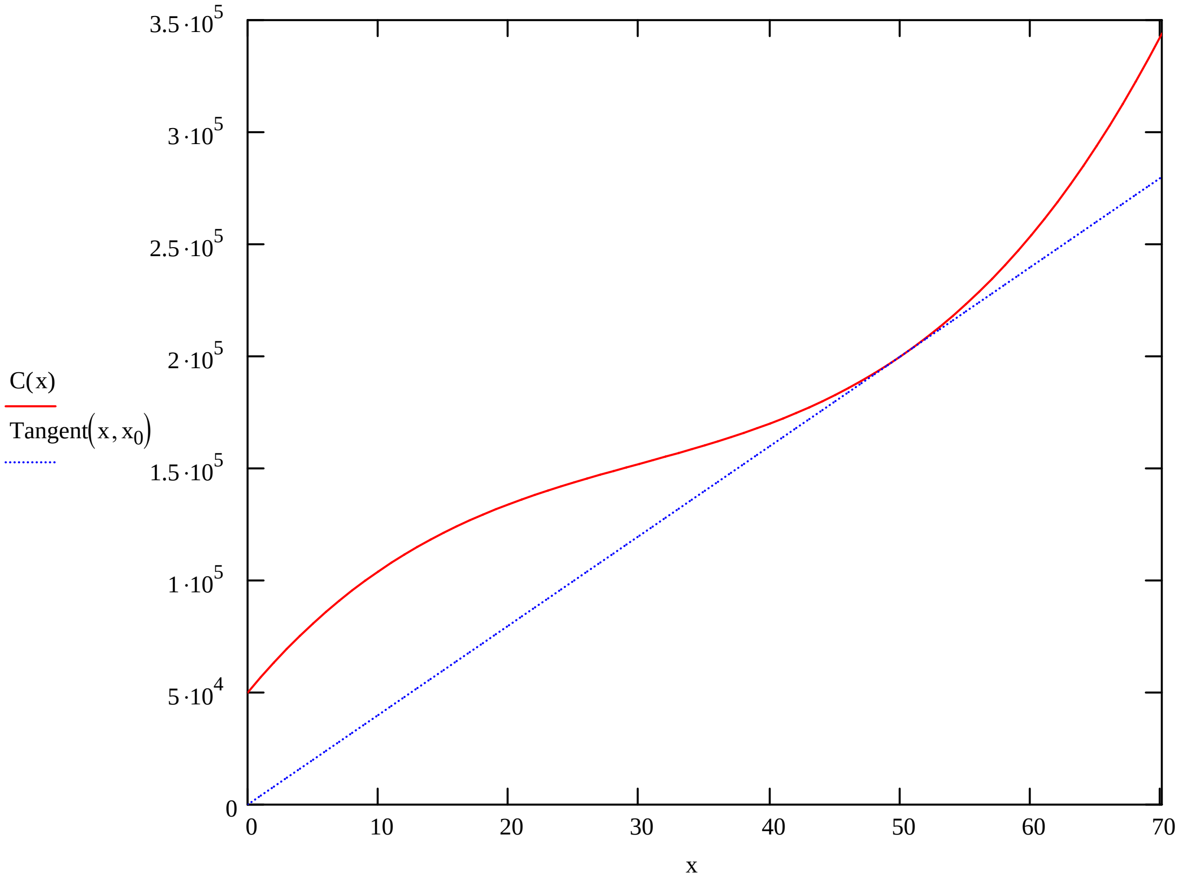


$x_0 := x_{cvmin}$



$x_0 := x_{cmin}$



5. Given are the following data:

$C_f := 2000$

$C_v(x) := 0.2x^2$

$C(x) := C_f + C_v(x)$

Which is the the value of c_v for $x_0 := 80$?

$c_v(x) := \frac{C_v(x)}{x}$

$x_0 = 80$

$x := x_0$

$c_v(x) = 16$

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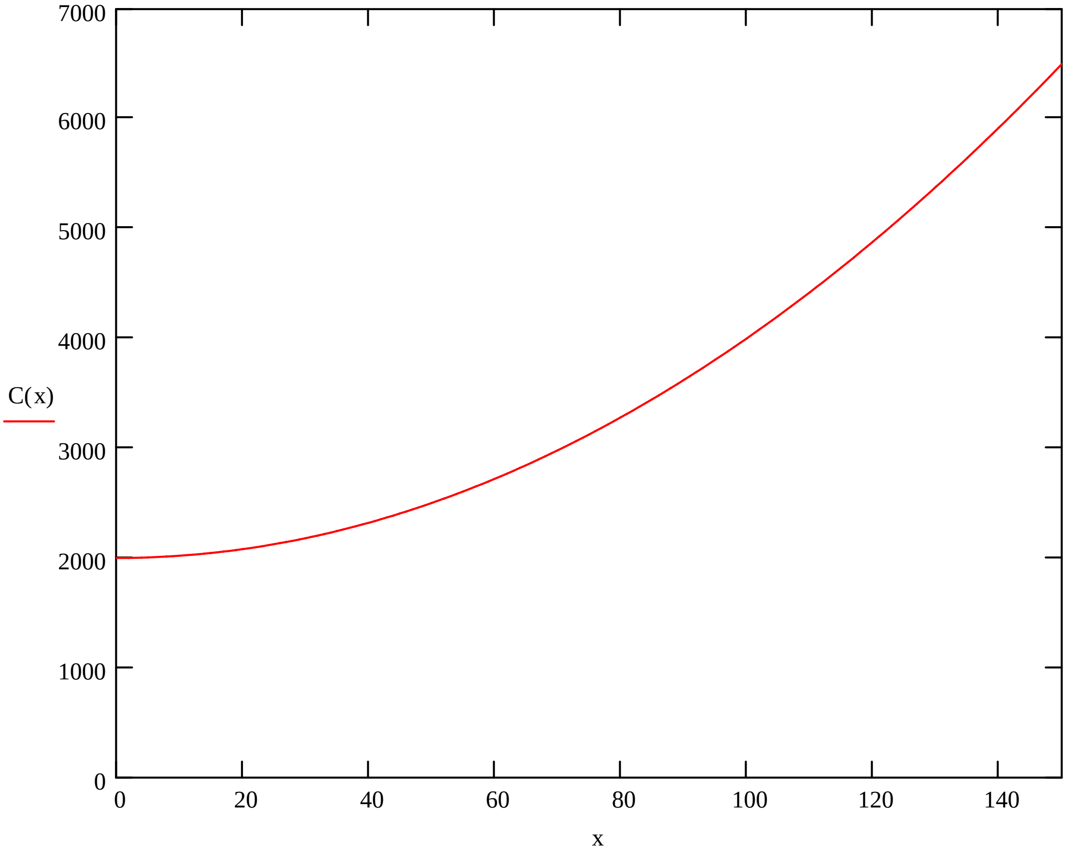
If $c_v(x_0)$ were regarded as a constant, what would the cost function look like? Use the following figure to draw this cost function.

$x := 0 .. 150$



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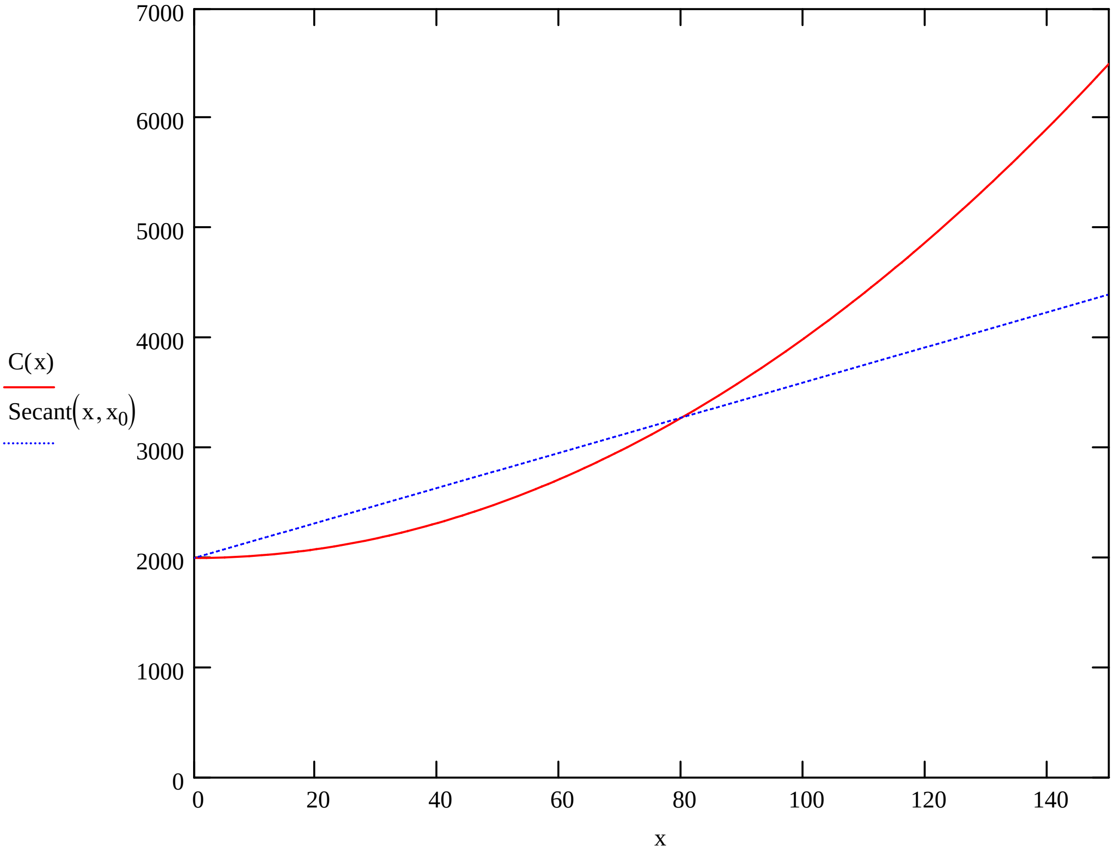
$$\mathrm{Secant}(x, x_0) := C_f + \frac{C_v(x_0)}{x_0} \cdot x$$



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6. Given are the following data:

$C_f := 2000$

$C_v(x) := 0.2x^2$

$C(x) := C_f + C_v(x)$

Which is the value of C' for $x_0 = 80$?

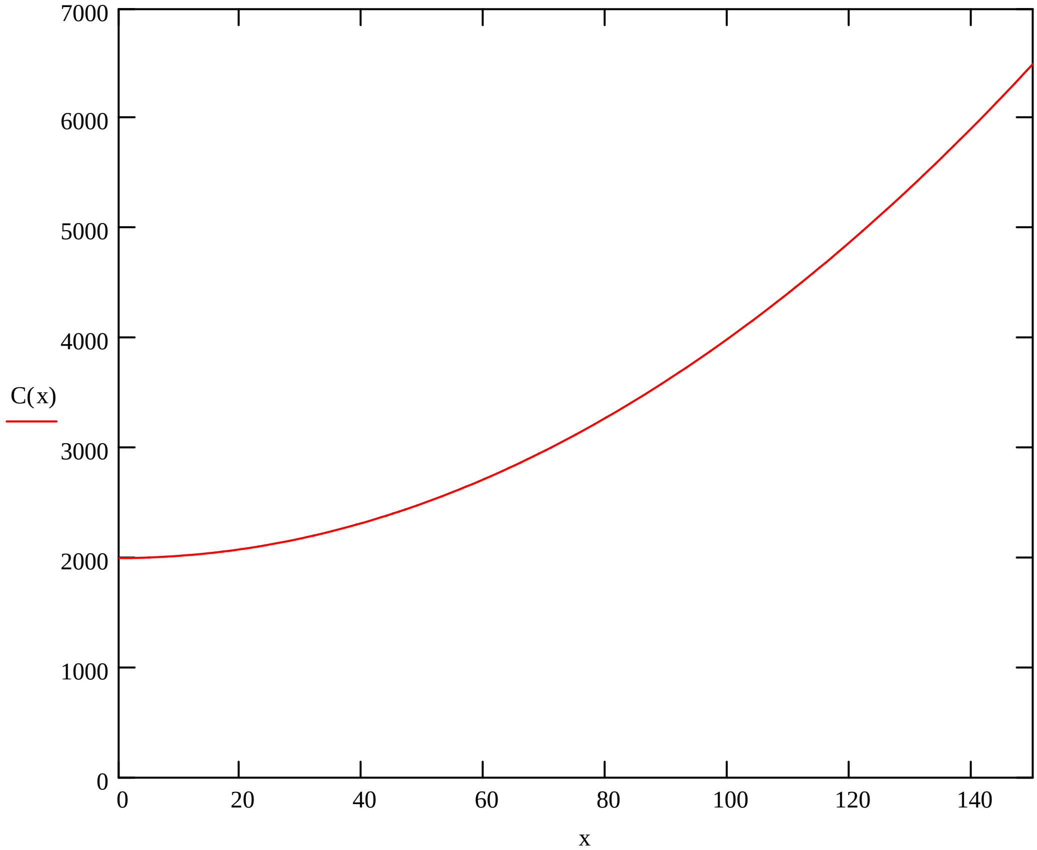
$C'(x) := 0.4x$

$x := x_0$

$C'(x_0) = 32$

If $C'(x_0)$ were regarded as a constant, what would the cost function look like? Use the following figure to draw this cost function.

$x := 0 .. 150$



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$$Tangent(x, x_0) := C(x_0) - C'(x_0) \cdot x_0 + C'(x_0) \cdot x$$